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Abstract

Financial institutions that are marketing tax-sheltered plans claim that the implied rates of return of tax-sheltered strategies are superior to those rates of return realized from taxable plans. The purpose of our paper is to investigate that claim. To accomplish our purpose, we have developed a model to determine, under different assumptions of various tax rates, the incremental benefits and the implied rates of returns of tax-deferred investments over the taxable investments. When the model is applied, the results show that tax-deferred investments are not always superior. Consequently, investors may not have a choice but to select portfolios at the lower end of the efficient frontier.

I. Introduction

Volume 28 Number 4 2005

Although many investors use strategies to defer tax liabilities, tax-deferred investments as an alternative investment strategy have attracted very limited academic research. Tax-deferral could be short-term or long-term. Shortterm strategies include shorting-against-the-box technique, purchasing a put option, or selling a deep-in-the-money call option. These strategies defer tax liabilities for a short period by locking-in relatively short-term capital gains of the underlying assets. Long-term tax-deferral is related to retirement plans. Substantial retirement-related research can be found in the informational materials provided by financial institutions that are marketing taxdeferred plans, and they often describe the most optimistic scenario. This paper deals only with long-term tax-deferred strategies and points out the circumstances where tax-deferral may not be as beneficial as advertised. The hypothesized superiority of the tax-deferred strategy is based on the tacit assumption that the post-retirement tax rate will be much lower than preretirement tax rate; hence the effective annual compounded rate of return from a tax-deferred plan is always superior to the rate of return from a taxable plan. The superiority of tax-deferred plan is based on additional assumption that in a taxable plan all investment gains are taxed at the marginal tax rate. In reality, the capital gain portion of return is taxed at a lower rate. This paper attempts to bridge these literature gaps by investigating whether or not taxdeferred investments are superior under generalized conditions.

Are Tax-Deferred Investments Invariably Superior?

The remainder of this paper is made up of five sections. Section II discusses the retirement planning process. Section III reviews the financial services literature on tax-deferred investments. In Section IV we derive a model that is subsequently used to test the hypothesis that the tax-deferred investment strategies are always superior to the taxable investment strategies. Section V discusses the results generated from applying the model and proffers some investment rules that would help a potential investor to decide whether or not to invest in tax-deferred or taxable investments. Section VI concludes the paper.

II. Retirement Planning Process

Retirement planning is a complex and time consuming but rewarding experience. The ultimate aim of the planning process is to achieve a financially secure and comparable life style in retirement. There are four basic planning steps associated with retirement. They are:

- (i) Project the retirement income that will allow the individual to maintain a desired life style. The rule of thumb often cited in popular literature is that 70%-80% of the pre-retirement income is needed to maintain similar pre- and post-retirement life styles.
- (ii) Anticipate the sources of income at retirement, e.g., Social Security benefits, pension benefits provided by the employer, and personal savings and investments.
- (iii) Project the value of and income from personal savings and investments.
- (iv) Determine the tax-deferred investments needed to achieve the desired income to achieve the estimated in step (i).

Non-academic financial literature on retirement-related investments published by financial institutions (business magazines, newspapers, tax guides, and retirement booklets) outline various tax-sheltered investment options available. These options are traditional IRA, Roth IRA, Keogh Plans, 401(k) and profit-sharing plans, 403(b) plans, deferred annuities, Employee Stock Ownership Plans (ESOPs), and Savings Incentive Match Plan for Employees (SIMPLE). While analyzing all these plans are beyond the scope of this research, this paper deals with the situation where pretax income is invested in a tax-deferred plan, thereby, taxes on investments and its earning are deferred till retirement, e.g., traditional IRAs and 403(b) plan.

III. Review of Literature

A number of academic articles on tax-deferred investments have been published in insurance and taxation journals (Mehr [1968], Healy [1981], Morehart and Trennepohl [1979], Adelman and Dorfman [1982], Gahin [1983] etc.). Most of these articles attest to the relative superiority of tax-deferred investment. However, some of the articles do point out that tax-deferred invest-



Management Research News

ments may not be superior if the marginal tax rate at retirement is higher than the pre-retirement tax rate. A good review of the literature can be found in Behzad, Lee and Vora (1993). Ragsdale, Seila, and Little [1994] do not specifically focus on the relative superiority of tax-deferred investments: it proffers an optimization model only for scheduling withdrawals from tax-deferred retirement accounts. Although unrelated to tax-deferred strategies, Butler and Domian [1993] computes probability distributions for retirement wealth over a range of investment horizons to develop asset returns over long holding periods in a form that would be useful for retirement planning.

Authors of personal finance and investment textbooks and reference books have adopted the position of financial institutions that are marketing tax-related products through highlighting their tax-saving advantages. They relied mainly on sources published in the non-academic journalistic financial media (for example, see Widicus and Stitzel, [1993], and Roy [1987]).

IV. Derivation of the Model

Volume 28 Number 4 2005

A review of the literature shows that only comparative cash flows have been used to promote the superiority of tax-sheltered plans as opposed to the non tax-sheltered plans under most favorable scenarios. We adopt similar methodology to examine tax-deferred investments under different scenarios. Further, we derive an expression for the incremental benefit of tax-deferral, and calculate the after-tax effective rate of return where an investor will be indifferent between taxable and tax-deferred investment. In this section, we develop a generalized model to determine the future value of the investment's accumulation and the implied effective annual rate of return under two possible investment scenarios, namely: taxable investment; and tax-deferred investment.

Let "A" denotes the after-tax amount which has been set aside for investment in a taxable fund subject to pre-retirement tax rate T_1 . Alternately, the pretax equivalent of the sum, $A/(1-T_1)$, is invested in a tax-deferred fund. This strategy insures maximum tax savings and maintains the level of take home income. The benefits of tax-deferral will be reduced if the invested amount is less than $A/(1-T_1)$.

Let the post retirement marginal tax rates be T_2 . Financial institutions marketing the tax-deferred investments assume that tax rate T_2 will be always lower than the current tax rate T_1 . This, however, may not always be true, since T_2 varies with both fiscal policy and political environment prevailing in the future. Although it is reasonable to assume that the post-retirement tax rate might be lower than the current rate, there are time-related conditions that would emerge in the future to keep the post-retirement tax rate unchanged or to make it even higher. Some of these conditions are: increase in other investment incomes; loss of deduction for ineligible grown-up dependAre Tax-Deferred Investments Invariably Superior?

ents; change in tax-payer status from "joint" to "single"; and loss of home mortgage interest deduction as mortgage is fully paid up.

Let k_1 denote the compounded annual rate of return on a taxable investment, and k₂ denote the compounded annual rate of return on taxdeferred investment. In promoting sales of tax-deferred products, brokerage firms claim that k_1 is invariably equal to k_2 , and consequently an investor would be better-off buying tax-deferred instruments, avoiding early settlement of their tax liabilities. Most investors, on the other hand, do not search and determine the opportunity cost of funds to be invested in the tax-deferred products. Contrary to this belief, we think that k_2 cannot exceed k_1 and sometimes it is less than k₁. Our premise stems from the fact that investors have limited choices on their investment menu when investing in tax-deferred vehicles. Their choices are confined to mutual funds, closed-end funds, life insurance-based fixed or variable annuities, or other portfolio whose composition is restricted due to management style and/or securities regulations. Investors, therefore, may be forced to choose portfolios at the lower-end of the efficient frontier, or they may be led to select even inferior, dominated sets of possibilities lying below the efficient curve. This self-inflicted choice is tantamount to engendering the benefit of the tax-deferred instrument over the taxable investment. Therefore, we assume the general case that the two rates, k_1 and k_2 , are not necessarily the same.

In existing literature, tax liability under a taxable plan is estimated at the marginal tax rate T_1 resulting in an after-tax rate of return of $k_1(1 - T_1)$. In general, the investment return comprises of interest income, dividend income, short-term capital gains distributed, long-term capital gains realized and distributed, and unrealized capital gains to be distributed in future. Interest and dividend incomes and short-term capital gain distributions are taxed at the marginal rate T_1 . Long-term capital gain distributions are taxed at the capital gains tax rate, T_{CG} .

Having discussed T_1 , T_2 , k_1 , and k_2 , our task now is to derive our model to help calculate the future values of a taxable investment, taxdeferred investment, and the tax-adjusted effective rate of return for the taxdeferred investment. Subsequently, the incremental benefits of tax-deferred investment are determined.

A. Taxable Investment Accumulation

Let "A" be the annual amount invested for "n" years in a taxable fund with a return of k_1 . The annual earnings from the investment are taxed at a rate T_1 and the remainder is reinvested. So, the after-tax rate of return is $k_1(1 - T_1)$, and the accumulated sum, or the future value FV₁ at the end of n years is defined as follows:



Management Research News

$$FV_{I} = A \times FVA_{k_{1}(1-T_{1}),n} = \frac{A[\{1+k_{1}(1-T_{1})\}^{n}-1]}{k_{1}(1-T_{1})}$$
(1)

where $FVA_{k_1(1-T_1),n}$ indicates the future value interest factor of an annuity compounded at the rate of $k_1(1-T_1)$ for "n" years. Note that there are no deferred tax liabilities on the accumulated amount at the end of "n" years.

B. Tax-deferred Investment Accumulation

According to this option, the pretax equivalent of the annual amount, $A/(1-T_1)$, is invested in a fund providing a rate of return of k_2 . Often, taxdeferred plans have limited investment opportunities compared to those investment opportunities available under the taxable plan. For example, investment in individual stocks, commodities, options and futures are not available for many tax-deferred plans. On the other hand, very few individuals would invest in fixed or variable annuities or life insurance-based plans under the taxable plan. Hence, k_2 does not necessarily have to be equal to k_1 . The taxdeferred accumulated sum, or the future value, FV_{II} , at the end of "n" years is defined as follows:

$$FV_{T} = \frac{A}{1 - T_{1}FVA_{k_{2},n}} = \frac{A}{1 - T_{1}} \frac{(1 + k_{2})^{n} - 1}{k_{2}}$$
(2)

$$NFV_{II} = (1 - T_2)FV_{II} = A \frac{1 - T_2}{1 - T_1} \frac{(1 + k_2)^n - 1}{k_2}$$
(3)

Now consider the case when there is no change in the tax rate at retirement, i.e., $T_1 = T_2$. In this case the net after-tax accumulation given in equation (3) is reduced to:

$$NFV_{II} = A[\frac{(1+k_2)^n - 1}{k_2}]$$
(4)

In this case, NFV_{II} is not a function of the marginal tax rates. For example, consider two individuals with current marginal tax rates of 28% and 36% respectively. Each individual is planning to replace their taxable investment of A per year with a tax-deferred investment. In order to maintain their consumption at current levels, they would invest A/(1-0.28) and A/(1-0.36), respectively, in a tax-deferred plan with an expected rate of return of k_2 . After paying the appropriate taxes at retirement at their marginal rates of 28% and 36%, both will end up with same net accumulation. Hence, if there is no reduction in the tax rate at retirement then tax-deferred investment will not provide greater benefit to the individual with a higher marginal tax rate. This is contrary to the common belief that individuals with higher marginal tax rates

Volume 28 Number 4 2005

Are Tax-Deferred Investments Invariably Superior?

49

are benefiting more from the tax-deferred investment. The greater benefit would be realized only if their marginal tax rate declines at retirement.

C. Incremental Benefits and Tax-adjusted Effective Rate

The incremental benefits, IB, associated with a tax-deferred investment over a taxable investment can be defined as follows:

$$IB = \frac{NFV_{II}}{FV_{I}} = \frac{(1 - T_{2})FV_{II}}{FV_{I}}$$
(5)

Substituting the values of FV_1 and FV_{II} from equations (1) and (2), we have:

$$IB = \frac{(1 - T_2) \frac{A}{(1 - T_1)} \frac{[(1 + k_2)^n - 1]}{k_2}}{\frac{A[\{1 + (1 - T_1)k_1\}^n - 1]}{k_1(1 - T_1)}}$$
$$= \frac{1 - T_2}{1 - T_1} \frac{k_1(1 - T_1)}{k_2} \frac{(1 + k_2)^n - 1}{\{1 + (1 - T_1)k_1\}^n - 1}$$
$$= \frac{1 - T_2}{1 - T_1} \frac{k_1(1 - T_1)}{k_2} \Phi(k_2; k_1, T_1, w)$$
(6)

The incremental benefit, IB, of tax deferral is dependent on four factors: (1) the difference between pre- and post-retirement tax rates, T_1 and T_2 ; (2) the relative rates of return of the tax-deferred and taxable funds, k_2 and k_1 ; (3) w, proportion of taxable return subject to lower capital gains tax rate; and (4) the compounding effect of the tax savings generated by investing in taxdeferred funds as determined by the function $\Phi(k_2; k_1, T_1, w)$. In most instances, investors are assumed to have a lower tax bracket in the postretirement period than in the pre-retirement period and the incremental benefit is usually greater than one. However, it is plausible to assume that there will be no reduction in the marginal tax rate at retirement or it may even increase (i.e., T_2 is greater than T_1).

When k_2 is greater than k_1 , the value of $\Phi(k_2; k_1, T_1)$ in equation (5) will be greater than one, implying superiority of tax-deferred investments. Unfortunately, it is extremely difficult to find an investment environment in which superior investment choices which are available for tax-deferred plans but not available for taxable plans, leading to k_2 being greater than k_1 . Invariably, the opposite is true.

On closer examination of equation (5), it is clear that investors should not invest in a tax-deferred plan if the rate of return of the tax-deferred plan is less than the after-tax rate of return of the taxable plan, i.e., $k_2 < (1-T_1)k_1$. To



Management Research News

conceptualize the significance of this constraint, let $k_2 = (1-T_1)k_1$. Consequently, the value of $\Phi(k_2; k_1, T_1)$ in equation (5) will be equal to one, and the incremental benefits, IB, is defined as follows:

$$IB = \frac{1 - T_2}{1 - T_1}$$

In this situation, the incremental benefit of tax-deferred investment will only be due to the difference between the pre- and post-retirement marginal tax rates. It should be emphasized that only under the condition, $k_2 > (1-T_1)k_1$, a tax-deferred investment may be superior to the taxable investment.

$$\frac{\{1+(1-T_1)k^*\}^n - 1}{(1-T_1)k^*} = \frac{1-T_2}{1-T_1} \frac{(1+k_2)^n - 1}{k_2}$$
(7)

Now we define tax-adjusted effective rates, k^* , as the indifference point between taxable and tax-deferred investments. For given values of T_1 , T_2 and k_2 , an investor would be indifferent between a tax-deferred plan and a taxable investment with a rate of return k^* , i.e., $IB(k^*; k_2; T_1; T_2) = 1$. Hence,

If $T_1=T_2$, then $k^* = k_2/(1-T_1)$. There is no closed-form solution for equation when T_1 is not equal to T_2 . However, it can be solved by iteration. An investor should switch from a taxable plan to a tax-deferred plan only if the rate of return from the taxable plan, k_1 is less than k^* . By doing so, the investor would minimize the opportunity loss and maximize his or her future accumulated earnings and return.

V. Application of the Model

Volume 28 Number 4 2005

Table 1 shows the taxable investments accumulation. Results in Table 1 indicate the effect of investment horizon and marginal tax rate on investor's after-tax accumulation. For example, if an investor with 28% marginal tax rates invests annually \$1,000 for 25 years at a compounded annual rate of return of 8%, it will grow to an accumulated value of \$53,046, compared to \$51,295 when the marginal tax rate is 31%.

Table 2 reports the after-tax net accumulation from the tax-deferred investment, NFV_{II}. It considers three possibilities for the future tax rates: (1) equal pre- and post- retirement tax rates $(T_1 = T_2)$, (2) a post-retirement tax rate that is greater than a pre-retirement tax rate $(T_2 > T_1)$ and (3) the most probable situation when post-retirement taxes rate less than a pre-retirement tax rate $(T_2 < T_1)$. The results show that, for given "n" and k₂, the net accumulated earnings NFV_{II} will be the same as long as $T_1=T_2$. If the rate of return, k₂, increases, the NFV_{II} will also increase. For example, an annuity of \$1,000 in-

Are Tax-Deferred Investments Invariably Superior?

vested for 25 years at a 12% return, it will grow to \$133,334 when $T_1=T_2$ (31%), or $T_1=T_2$ (28%). However, the accumulated sum is slightly greater than the corresponding sum in Table 1. When $T_2>T_1$ (31% vs. 28%), the NFV_{II} is lower than the corresponding values in Table 1 with 5 and 10 year investment horizons but higher for 15, 20 and 25 year investment horizons. Furthermore, *ceteris paribus*, the higher the return the higher the NFV_{II}. The real benefit of tax-deferred investment is when $T_2>T_1$

Table 3 presents the incremental benefits, IB, of tax-deferred investment when pre- and post-retirement tax rates are equal to 28% and 20-year investment horizons. It is clear that if $k_2 < k_1$, then the tax-deferred investment may not always be superior. Some tax-deferred investment providers charge account maintenance fees, limit the choice of assets and restrict the use of return-enhancing strategies such as margin buying and option trading. Consequently, an investment in tax-deferred account may net lower return than an identical investment in a taxable account. Table 4 reports the incremental benefits, IB, of tax-deferred investment for different tax scenarios and net return of the tax-deferred account. It reinforces the conclusions from Table 3 and identifies the situations where tax-deferred investments are not superior. These cases represent an opportunity loss with tax-deferral.

The tax-adjusted effective rates, k^* , of a tax-deferred plan under various tax scenarios are reported in Table 5. Let us consider the case when the current tax rate of 28% is not reduced at retirement. For a 20-year investment horizon, investors would be indifferent between 8% tax-deferred investment and a taxable investment with 11.11% return. In this case $k^* = k_2/(1-T_1)$. Consider the case when the tax rate is reduced from 28% to 15%. For a 20-year investment horizon, investors would be indifferent between 8% tax-deferred investment with return greater than k^* is preferred to the appropriate tax-deferred investment In Table 5.

Figure 1 compares net accumulations from the taxable investment versus tax-deferred investment. Investor's opportunity cost of funds is 12%. A 12% rate of return is easily attainable in the taxable environment by investing in a portfolio of stocks or stock mutual fund. In this case, the individual would make the appropriate investment decision and would invest in a tax-deferred plan with 12% expected rate of return. The benefit of tax-deferral is quite obvious. Figure 2 compares the taxable and an inferior tax-deferred plan. An investor with 12% opportunity cost invests in a tax-deferred plan with 8% rate of return, i.e., life insurance-based products or fixed income funds. When there is no reduction in the marginal tax rate at retirement, there is an opportunity loss associated with tax-deferral. With a reduction of marginal tax rates at retirement, there is a very small benefit in tax-deferred strategy.



Management Research News

VI. Conclusions

Volume 28 Number 4 2005

This paper derives a model that would determine the incremental benefits and the implied rates of returns of tax-deferred investments. Based on the above model, we would like to recommend some investment rules for tax-deferred investments. An investor should be cognizant of the fact that tax-deferred investments are not always superior, considering the continued changes in tax rates. Forecasting the future individual income tax rates is a complex process, given the associated complex political and fiscal uncertainties. Consequently, an investor may like to employ multiple- scenario analysis to examine the tax rates that might prevail in the future. Examination of all tax options and the associated expected rates of return is very important. Of particular importance is to determine whether the tax rate at retirement is indeed going to be less than the pre-retirement tax rate. Are Tax-Deferred Investments Invariably Superior?

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Are Tax-Deferred Investments Invariably Superior?

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Management Research News

Taxable Investment Accumulation $F_{k_1} = \frac{A[\{1 + k_1(1 - T_1)\}^n - 1]}{k_1(1 - T_1)}$								
	Holding	Taxable	Investment	Return				
	Period	k ₁ =5%	k ₁ =8%	k ₁ =12%				
	5 years	\$ 5,357	\$ 5,583	\$ 5,899				
Pre-Retirement	10 years	11,704	12,887	14,681				
Tax Rate,	15 years	19,225	22,443	27,751				
$T_1 = 31\%$	20 years	28,135	34,943	47,206				
	25 years	38,692	51,295	76,165				
	5 years	5,373	5,610	5,942				
Pre-Retirement	10 years	11,786	13,033	14,934				
Tax Rate,	15 years	19,439	22,855	28,543				
$T_1 = 28\%$	20 years	28,572	35,850	49,139				
	25 years	39,472	53,046	80,308				
	5 years	5,443	5,728	6,129				
Pre-Retirement	10 years	12,146	13,687	16,091				
Tax Rate,	15 years	20,400	24,745	32,281				
T ₁ = 15%	20 years	30,563	40,111	58,592				
-	25 years	43,076	61,462	101,354				

Annual investment in taxable plan = \$1,000. All earning paid out and taxed at the marginal tax rate.

Are Tax-Deferred Investments Invariably Superior?

Volume 28 Number 4 2005



Table 2Net Accumulation of Tax-Deferred Investment $NFV_{II} = A \frac{(1-T_2)}{(1-T_1)} \frac{(1+k_2)^n - 1}{k_2}$										
	Holding Period	No Change in Tax Rate T ₁ =T ₂ =31 %	No Change in Tax Rate T ₁ =T ₂ =28 %	Modest Increase in Tax Rate T ₁ =28, T ₂ =31%	Large De- crease in Tax Rate T ₁ =28, T ₂ =15%					
	5 years	\$ 5,526	\$ 5,526	\$ 5,295	\$ 6,523					
Tax-deferred	10 years	12,578	12,578	12,054	14,849					
Investment Return	15 years	21,579	21,579	20,679	25,475					
$k_2 = 5\%$	20 years	33,066	33,066	31,688	39,036					
	25 years	47,727	47,727	45,738	56,344					
	5 years	5,867	5,867	5,622	6,926					
Tax-deferred	10 years	14,487	14,487	13,883	17,102					
Investment Return	15 years	27,152	27,152	26,021	32,055					
$k_2 = 8\%$	20 years	45,762	45,762	43,855	54,025					
	25 years	73,106	73,106	70,060	86,036					
	5 years	6,353	6,353	6,088	9,580					
Tax-deferred	10 years	17,549	17,549	16,818	24,384					
Investment Return	15 years	37,280	37,280	35,726	44,011					
$k_2 = 12\%$	20 years	72,052	72,052	69,050	85,062					
	25 years	133,334	133,334	127,778	157,408					

Pre and post-retirement tax rates are T_1 and T_2 respectively. Annual investment in taxable plan = \$1,000 and in tax-deferred plan = \$1,000/(1- T_1).



Are Tax-Deferred Investments Invariably Superior?

Management Research News

	15%	.351	.387	.428	.475	.527	.586	.653	.729	.815	.913	1.023	1.148	1.290	1.451	1.633	.(¹].	re Tax-De Invest	
	14%	.381	.420	.465	.515	.572	.637	.709	.792	.885	166.	1.111	1.247	1.401	1.575	1.772	\$1,000/(1-T ₁).	Inva	
	13%	.413	.456	.504	.559	.621	169.	.770	.859	096.	1.075	1.205	1.353	1.519	1.709	1.923			
	12%	.448	.495	.547	909.	.673	.749	.834	.931	1.041	1.166	1.307	1.466	1.647	1.852	2.085	x-deferre		
	11%	.485	.536	.592	.657	.729	.811	.904	1.009	1.128	1.263	1.416	1.589	1.785	2.007	2.259	S1,000 and in tax-deferred plan =		
ment	10%	.526	.580	.641	.711	.789	.878	.978	1.092	1.221	1.367	1.532	1.720	1.932	2.172	2.445	11		
ed Invest $\frac{i-1}{ k_i ^n-1}$	9%	.568	.627	.694	.769	.854	.950	1.058	1.181	1.321	1.478	1.657	1.860	2.089	2.350	2.644	xable plai		
ble 3 Fax-Deferred 1 $(1+k_2)^n - 1$ $\{(1+(1-T_1)k_1)^n$	8%	.614	.668	.750	.831	.922	1.026	1.144	1.277	1.427	1.598	1.791	2.010	2.258	2.539	2.858	ment in ta		
Table 3 enefit of Tax-] $-T_2)(\frac{k_1}{k_2})\frac{(1)}{(1+1)}$	70/0	.663	.732	.809	768.	966.	1.108	1.235	1.378	1.541	1.725	1.934	2.170	2.438	2.741	3.085	ual invest		
Table 3 Incremental Benefit of Tax-Deferred Investment $IB = (1 - T_2) \left(\frac{k_1}{k_2}\right) \frac{(1 + k_2)^n - 1}{(1 + (1 - T_1)k_1)^n - 1}$	6%0	.715	.789	.873	.967	1.074	1.195	1.332	1.486	1.662	1.860	2.085	2.340	2.629	2.957	3.328	$T_1 = T_2 = 28\%$. Annual investment in taxable plan		
Increm	5%	177.	.850	.940	1.042	1.157	1.288	1.435	1.602	1.791	2.005	2.247	2.522	2.833	3.186	3.585	e, T ₁ =T ₂ =		
	4%	.830	.915	1.012	1.122	1.246	1.386	1.544	1.724	1.927	2.158	2.419	2.714	3.049	3.429	3.859	in tax rate,		
	3%	.892	.984	1.088	1.206	1.339	1.490	1.661	1.854	2.072	2.320	2.601	2.919	3.279	3.687	4.150	Vo change		
	2%	.958	1.057	1.169	1.295	1.438	1.600	1.783	1.991	2.226	2.492	2.793	3.134	3.521	3.960	4.457	20 years. 1		
	1%	1.028	1.134	1.254	1.390	1.543	1.717	1.913	2.136	2.387	2.673	2.996	3.362	3.778	4.248	4.781	Holding period is 20 years. No ch		
	k , k2	1%	2%	3%	4%	5%	6%0	70%	8%	9%	10%	11%	12%	13%	14%	15%	Holding 1		

57

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Volume 28 Number 4 2005

Inci		Table 4 tefit of Tax-D T_2)($\frac{k_1}{k_2}$) (1- (1+(1+(1+(1+(1+(1+(1+(1+(1+(1+(1+(1+(1+(stment	
Opportunity cost of fu	nds: k ₁ = 12% Holding Period	No Change in Tax Rate T ₁ =T ₂ =31 %	No Change in Tax Rate T ₁ =T ₂ =28 %	Modest Increase in Tax Rate T ₁ =28, T ₂ =31%	Large Decrease in Tax Rate T ₁ =28, T ₂ =15%
	5 years	0.937	0.930	0.891	1.098
Tax-deferred	10 years	0.857	0.842	0.807	0.994
Investment Return	15 years	0.776	0.756	0.725	0.893
$k_2 = 5\%$	20 years	0.700	0.673	0.645	0.794
	25 years	0.625	0.594	0.570	0.702
	5 years	0.995	0.987	0.946	1.166
Tax-deferred	10 years	0.987	0.970	0.930	1.145
Investment Return	15 years	0.978	0.951	0.912	1.123
$k_2 = 8\%$	20 years	0.969	0.931	0.893	1.099
	25 years	0.960	0.910	0.872	1.075
	5 years	1.077	1.069	1.025	1.262
Tax-deferred	10 years	1.195	1.175	1.126	1.387
Investment Return	15 years	1.343	1.306	1.252	1.542
$k_2 = 12\%$	20 years	1.526	1.466	1.405	1.731
		1			

25 years Pre and post-retirement tax rates are T_1 and T_2 respectively.

Tables with other values of k_1 are available to interested readers.

Annual investment in taxable plan = 1,000 and in tax-deferred plan = $1,000/(1-T_1)$.

1.751

1.660



Management Research News

1.591

1.960

Tax-Defe			$\frac{1}{T_2} \frac{(1+k_2)^n}{k_2}$		*
	Holding Period	No Change in Tax Rate T ₁ =T ₂ =31 %	No Change in Tax Rate T ₁ =T ₂ =28 %	Modest Increase in Tax Rate T ₁ =28, T ₂ =31%	Large Decrease in Tax Rate T ₁ =28, T ₂ =15%
	5 years	7.25%	6.94%	3.99%	18.52%
Tax-deferred	10 years	7.25%	6.94%	5.67%	11.83%
Investment Return	15 years	7.25%	6.94%	6.16%	9.97%
$k_2 = 5\%$	20 years	7.25%	6.94%	6.38%	9.10%
	25 years	7.25%	6.94%	6.51%	8.60%
	5 years	11.59%	11.11%	8.05%	22.72%
Tax-deferred	10 years	11.59%	11.11%	9.86%	15.93%
nvestment Return	15 years	11.59%	11.11%	10.35%	14.07%
$k_2 = 8\%$	20 years	11.59%	11.11%	10.58%	13.17%
	25 years	11.59%	11.11%	10.71%	12.68%
	5 years	17.39%	16.67%	13.69%	28.33%
Tax-deferred	10 years	17.39%	16.67%	15.44%	21.41%
nvestment Return	15 years	17.39%	16.67%	15.93%	19.51%
$k_2 = 12\%$	20 years	17.39%	16.67%	16.16%	18.64%
	25 years	17.39%	16.67%	16.28%	18.16%

Pre- and post-retirement tax rates are T_1 and T_2 respectively.

Volume 28 Number 4 2005

Annual investment in taxable plan = \$1,000 and in tax-deferred plan = $\$1,000/(1-T_1)$. Taxable investment with return greater than k^* is preferred over the tax-deferred investment.



59

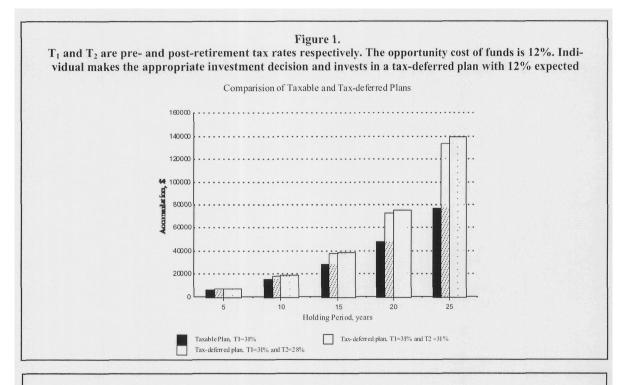
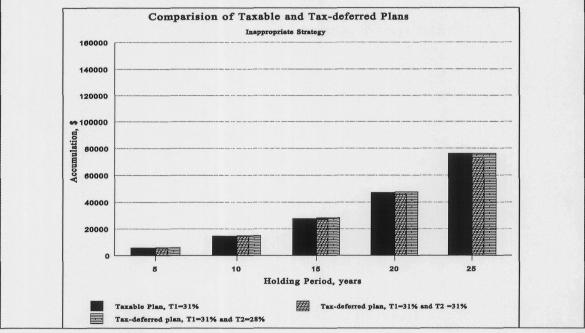


Figure 2.

T₁ and T₂ are pre- and post-retirement tax rates respectively. The opportunity cost of funds is 12%. Individual makes the appropriate investment decision and invests in a tax-deferred plan with 8% expected rate of return.



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